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# A REACH SET MPC SCHEME FOR THE COOPERATIVE CONTROL OF AUTONOMOUS UNDERWATER VEHICLES

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## Abstract

A Model Predictive Control (MPC) based architecture is discussed to address the problem of the coordinated control of Autonomous Underwater Vehicles in an environment in which not only the acoustic communications and perturbations impose formidable challenges but also other two important classes of issues are considered: (i) unexpected emergence of obstacles, and (ii) severe onboard computation constraints. The later aspect is discussed by a novel implementation of the MPC scheme - the Reach Set MPC - whose underlying formal framework is briefly outlined. Finally some simulation results are presented.

## Key words

Coordinated control; motion formation; model predictive control; obstacle avoidance; reach set

## 1 Introduction

This article concerns the investigation of an efficient Model Predictive Control type of scheme satisfying the strict requirements arising in multi-agent cooperative control of systems of autonomous robotic vehicles such as underwater vehicles (AUVs), unmanned air vehicles (UAVs), unmanned ground vehicles (UGVs) and spacecrafts.

The issues motivating the development of such systems are extremely pervasive as they permeate most of the great challenges in most of the human endeavors, from advanced manufacturing systems to the advancement of the badly needed knowledge concerning all terrestrial (here, environment and climate have been strongly emphasized) and extra-terrestrial processes, and passing by the management of all human processes and of extraction of resources. More precisely, climate change, biodiversity, environment, natural resources management, territory management, mobility, security and surveillance, to name just a few, are some of the real world challenges that, today, human kind is perceiving as extremely urgent impose a number of increasingly sophisticated requirements

for the underlying field studies data gathering, [Paley et al., 2008; Zhang et al., 2007; Fiorelli et al., 2004]. Spatial and temporal distribution, persistence, combination of wide area with local area data sampling, etc., are some general requirements calling for a concerted instrumentation of the earth which encompasses fixed, mobile sensor platforms and other devices networked together. Many of these systems fall within the so-called distributed networked cyberphysical systems. One can easily devise many instances of missions (i.e., set of activities with a pre-defined purpose) involving, possibly heterogeneous, networked unmanned vehicles among which different sensors need to be distributed that should move in a concerted way defined to fulfill the specified data sampling requirements. Thus, the need for the motion cooperation of multiple robotic vehicles surely raises key issues in the design of such systems.

Thus, it is not surprising that extensive research on multi-agent cooperative control with autonomous systems has been conducted for some time now. Systems such as autonomous underwater vehicles (AUVs), unmanned air vehicles (UAVs), spacecrafts, ground vehicles or robots are the most considered. The range of AUV applications include seabed imaging and mapping, gradient search and lost cargo detection. Spacecraft formation flying is required for applications such as the monitoring of Earth and its surrounding atmosphere, geodesy, deep space imaging and exploration, and in-orbit servicing and maintenance of spacecraft. Also UAVs and AUVs have been widely used for inspection and monitoring of systems and infrastructures (oil and gas platforms, port facilities, energy parks, among other assets), ground surveying and mapping (accident or fire detection, military reconnaissance), and cargo transportation.

Naturally, formidable control and estimation challenges are posed by these systems's nature and high extent of autonomy combined with complex non-linear coupled dynamics, sophisticated environmental perception - e.g., obstacle avoidance - for situational awareness, as well as, the management of on-

board scarce resources required for propulsion, computation, sensing and communications. These general requirements call for an increasing incorporation for optimization-based techniques, notably the state feedback control scheme designated by MPC, in increasingly complex control design that, over the years, have been proposed in many approaches.

The research documented in this article is a step forward in this context. Here, we embed the MPC scheme in an overall architecture that, by taking into account the onboard sensing and computation capabilities, allows the synthesis of control strategies preventing the collision with obstacles while promoting the optimization of the overall resources management. Moreover, we improve on the state-of-the-art in the sense that a novel MPC implementation maximizing the amount of off-line pre-computation whenever this allowed by time-invariant information.

This article is organized as follows: In the next section we provide overview of the most pertinent state-of-the-art. In the following section, we present the overall control problem formulation with emphasis in the key challenging requirements. Then, in section 4, we present the computationally efficient MPC scheme for the decentralized cooperative control of multiple AUVs. In section 5, we present the overall architecture that articulates the MPC scheme with the sensing capabilities to avoid collision with obstacles. Preliminary results are shown in section 6 just before the last section that includes some conclusions and future work.

## 2 Brief State-of-the-Art

The control problem of vehicles in fluids addressed in this article is difficult. The system not only exhibits strong nonlinearities and is subject to significant uncertainties in its parameters and is subject to strong perturbations, but also is, in fact, of distributed parameter nature. A very good reference is [Fossen, 1994].

Various types of controllers from non-linear control theory have been popular in this domain. [Kristiansen and Nicklasson, 2009] presents a review of existing methods on tight spacecraft formation flying that use state feedback. In [Ren and Beard, 2002], an approach for formation control using a virtual structure is proposed. A back-stepping controller which is robust to input constraints and parameter uncertainties for spacecraft formation is developed in [Lv et al., 2011].

MPC is a well-known time domain control strategy that computes control inputs by solving finite horizon open-loop optimal control problems in a receding horizon fashion. Because of its optimization, MPC provides a rather flexible framework to digest complicated system dynamics, and to incorporate intractable constraints. The importance of optimization in AUVs formation control is acknowledged in [Breger et al., 2005] where an MPC based formation controller with sensing noise is developed. MPC was been widely investigated in [Mayne et al., 2000] where several applications were addressed. Underwater communication are

difficult and such constraints for AUV formation control were thoroughly addressed in [Franco et al., 2004; Franco et al., 2008; Keviczky et al., 2006; Keviczky et al., 2008; Fax and Murray, 2004; Saber and Murray, 2004; Kazerooni and Khorasani, 2008; Goodwin et al., 2004; Fontes et al., 2009; Gruene et al., 2009; Allen et al., 2002; Liu et al., 2001]. The problem of cooperative control of a team of distributed agents with decoupled nonlinear discrete-time dynamics and exchanging delayed information is addressed in [Franco et al., 2004; Franco et al., 2008]. Building on the work reported in [Keviczky et al., 2006], a decentralized scheme for the coordinated control of formations of autonomous vehicles is presented in [Keviczky et al., 2008]. If feasibility of the decentralized control is lost, collision avoidance is ensured by invoking emergency maneuvers that are computed via invariant set theory. A stabilization analysis can be found in [Keviczky et al., 2006].

In [Consolini et al., 2008] a leader-follower formations of nonholonomic mobile robots is presented, in which the control inputs are forced to satisfy suitable constraints that restrict the set of leader possible paths and admissible positions of the follower with respect to the leader. [Ghommam et al., 2008] presents a virtual structure control strategy for the coordination of multiple mobile robots using unicycle model. Other types of formation controllers using MPC and fast-marching methods have also been developed. In [Liang and Lee, 2006], the problem of formation control and obstacle avoidance for a group of nonholonomic mobile robots using MPC is considered.

A nonlinear model predictive control (NMPC) framework for collision-free formation flight controller design for unmanned aerial vehicles was proposed in [Chao et al., 2011] being the formation flight controller designed in a distributed way. More recently, we found applications of output-feedback MPC [Quintero et al., 2015] where the problem of two UAVs tracking an evasive moving ground vehicle is solved, and a comprehensive framework for the cooperative guidance of fleets of autonomous vehicles relying on MPC and addressing subjects as collision and obstacle avoidance, formation flying and area exploration [Bertrand et al., 2014]. Path tracking model predictive control of a tilt-rotor UAV carrying a suspended load is developed in [Andrade et al., 2016], and the path following control of an AUV using multi-objective model predictive control, or the nonlinear model predictive control for trajectory tracking of an AUV [Shen et al., 2016].

However, all these approaches suffer from several key drawbacks for AUVs: (i) computationally intensive nature of the MPC scheme that involves solving recursively a sequence of optimal control problems with very limited onboard computation capabilities and energy; and (ii) the MPC schemes are parameterized in order to ensure convergence and stability and, in general, they are not related to onboard sensing capabilities. Our approach addresses these issues in this article.

### 3 Problem formulation

The AUV formation control problem consists in controlling a set of AUVs to track a trajectory while maintaining a formation under constraints on the state, control and communications. We consider the usual model, [Fossen, 1994] with coefficients based on the results from [Prestero, 2001] and from our own field experiments:

$$\dot{\eta} = \begin{bmatrix} u \cos(\psi) - v \sin(\psi) \\ u \sin(\psi) + v \cos(\psi) \\ r \end{bmatrix},$$

$$\dot{\nu} = \begin{bmatrix} \frac{\tau_u - (m - Y_{\dot{v}})vr - X_{u|u}|u|}{m - X_{\dot{u}}} \\ \frac{(m - X_{\dot{u}})ur - Y_{v|v}|v|}{m - Y_{\dot{v}}} \\ \frac{\tau_r - (Y_{\dot{v}} - X_{\dot{u}})uv - N_{r|r}|r|}{I_{zz} - N_{\dot{r}}} \end{bmatrix},$$

where  $\eta = [x, y, \psi]^T$ ,  $\nu = [u, v, r]^T$ ,  $\tau = [\tau_u, \tau_r]$ , the coefficients  $X_{\dot{u}}$ ,  $Y_{\dot{v}}$ ,  $N_{\dot{r}}$  represents hydrodynamic added mass,  $X_{u|u}$ ,  $Y_{v|v}$ ,  $N_{r|r}$  the hydrodynamic drag and  $m$  the vehicle mass.

From the above, we are interested in control strategies which, for each AUV  $i$ ,  $i = 1, \dots, n_v$ , minimize, over a given time interval, the cost functional with two terms, one that penalizes the trajectory tracking error forcing vehicles to follow the desired path,  $\eta_r^i$ , and another that penalizes the control effort, thus saving the limited energy on board of vehicles, i.e.,

$$\int_t^{t+T} [(\eta^i(s) - \eta_r^i(s))^T Q (\eta^i(s) - \eta_r^i(s)) + \tau^i(s)^T R \tau^i(s)] ds,$$

where  $\eta^i$  and  $\tau^i$  are satisfied kinematic and dynamic constraints above, the endpoint state constraints,  $\eta^i(t+T) \in C_{t+T}$ , the control constraints,  $\tau^i(s) \in \mathcal{U}^i$ , the state constraints,  $(\eta^i(s), \nu^i(s)) \in \mathcal{S}^i$ , the communication constraints  $g_{i,j}^c(\eta^i(s), \eta^j(s)) \in C_{i,j}^c, \forall j \in \mathcal{G}^c(i)$ ; and the formation constraints  $g_{i,j}^f(\eta^i(s), \eta^j(s)) \in C_{i,j}^f, \forall j \in \mathcal{G}^f(i)$ . For additional details, see [Gomes et al., 2011].

There is a vast body of literature on MPC, [Mayne et al., 2000]). This is a control scheme in which the control action for the current time subinterval – control horizon – is obtained, at each sampling time, by solving on-line an optimal control problem over a certain large time horizon – the prediction horizon – with the state variable initialized at the current best estimate updated with the latest sampled value. Once the optimization yields an optimal control sequence, this is applied to the plant during the control horizon. Then, once this time interval elapses, the state is sampled and the process is re-iterated. The MPC scheme involves the following steps:

1. Initialization. Let  $t_0$  be the current time, and set up the initial parameters or conditions specifying, possibly among others,  $x_0$ ,  $T$ ,  $\Delta$ , initial filter parameters.
2. Sample the state variable at time  $t_0$ .
3. Compute the optimal control strategy,  $u^*$ , in the prediction horizon, i.e.,  $[t_0, t_0 + T]$ , by solving the optimal control problem (P).

4. Apply the obtained optimal control during the current control horizon,  $[t_0, t_0 + \Delta]$ .
5. Slide time by  $\Delta$ , i.e.,  $t_0 = t_0 + \Delta$ , and adapt parameters and models as needed.
6. Go to step 2.

where  $x_0$  is the initial state,  $T$  is the prediction horizon for control optimization, and  $\Delta$  is the control horizon. Two key variants to this scheme are important for networked systems implementation: (i) the data obtained in step 4. usually is a composition of locally sampled data and data communicated from other vehicles or subsystems, and, thus, it might be of interest to replace the data that failed to be transmitted by simulated data; (ii) To address communication delays and missed data, either by replacing the absent data by simulated one, or MPC parameters may be adjusted. A typical general formulation of the optimal control problem (P) is as follows:

$$(P) \text{ Minimize } g(x(t_0 + T)) + \int_{t_0}^{t_0+T} f_0(t, x(t), u(t)) dt$$

subject to  $\dot{x}(t) = f(t, x(t), u(t)) \quad \mathcal{L} - a.e.$

$u(t) \in \Omega \quad \mathcal{L} - a.e.$

$h(t, x(t)) \leq 0$

$g(t, x(t), u(t)) \leq 0$

$x(t_0 + T) \in C_f$

where  $g$  is the endpoint cost functional,  $f_0$  is the running cost integrand,  $f$ ,  $h$ , and  $g$  represent, respectively, the vehicle dynamics, the state constraints, and the mixed constraints,  $C$  is a target that may also be specified in order to ensure stability. If one wants to take into account the uncertainty with respect to the initial state, then one may consider an initial state constraint, i.e.,  $x(t_0) \in C_i$  where  $C_i$  is an estimate of the uncertainty set, being the minimization taken over the worst case of the initial state. For a discuss stability and robustness, [Mayne et al., 2000; Langson et al., 2004; Mayne et al., 2009]. However, problem (P) is too computationally intensive for the onboard computational resources, power consumption constraints, as well as real-time constraints. However, this difficulty is overcome by replacing the optimal control problem (P) by a much smaller finite dimensional optimization that will be described in the next section.

The decentralized character of the overall MPC controller is due to the fact that each vehicle runs its own MPC scheme (which encompasses the models of its neighboring AUVs) and communicates only with its neighbors. Moreover, communication delays and packet dropouts as well as noise and disturbances can easily be incorporated. Finally, in our decentralized framework, each AUV runs the same type of controller.

### 4 Reach set MPC

The reachable set based MPC scheme we consider here is in the context of approximating a long (possibly infinite) time horizon optimization problem by a

sequence of sliding shorter time horizon finite dimensional optimization sub-problems initialized with the current sampled state. The basic idea consists in replacing the infinite dimensional optimization problem by a sequence of finite dimensional ones. This requires two items: (i) propagation in time of the cost functional in order to ensure consistency; and (ii) forward propagation of the reach set starting on the current value of the sampled state variable. Notice that this formulation of the optimization problem exhibits all the advantages inherent to the geometric character, namely in what concerns the incorporation of additional constraints as well as uncertainties in the dynamics.

This also enables to close the control loop since the sampled state also reflects the effect of perturbations in the evolution of the state trajectory. Moreover, this scheme enables the incorporation of features of the environment - e.g., potential static or dynamic obstacles arising within the usually limited sensors detection range - that are not present in the conventional optimal control formulation, and, thus, in the usual associated MPC schemes, but that are quite natural for many application scenarios such as those involving autonomous robotic vehicles. The key general issue underlying this novel MPC scheme is to pre-compute off-line all the ingredients required for the on-line (feedback synthesis) that remain invariant in the course of the “mission execution”. The next important issue involved in this scheme consists in the mechanisms to adjust the involved ingredients: the Reach Set at each  $(t, x)$  and the short cost functional to be considered and that approximates the overall cost functional. These two ingredients are considered next.

#### 4.1 Short term “equivalent” cost functional and Reach Set

First, we consider the optimization of a dynamic control system over a very long time horizon  $[0, T]$ , with, possibly  $T = \infty$ ,

$$(P_{T_f}) \text{ Minimize } g(x(T_f)) + \int_0^{T_f} l(t, x(t), u(t)) dt$$

$$\text{subject to } \dot{x}(t) = f(t, x(t), u(t)), \quad \mathcal{L} - a.e.$$

$$x(T_f) \in C_f, \quad x(0) \text{ is given, } u \in \mathcal{U},$$

where  $\mathcal{U} := \{u : [0, T_f] \rightarrow \mathbb{R}^m : u(t) \in \Omega\}$ , with  $\Omega$  closed. For  $T = \infty$ , we consider trajectories converging asymptotically to some equilibrium point. For  $T < T_f$ , the Principle of Optimality implies that the solution to  $(P_{T_f})$  restricted to the interval  $[t, t+T]$  is also a solution to  $(P_T)$  below.

$$(P_T) \text{ Min } V(t+T, x(t+T)) + \int_t^{t+T} l(\tau, x(\tau), u(\tau)) d\tau$$

$$\text{s. t. } \dot{x}(\tau) = f(\tau, x(\tau), u(\tau)), \quad \mathcal{L} - a.e.$$

$$u \in \mathcal{U}, \text{ and } x(t) \text{ is given,}$$

where  $V(t, z) := \min_{u \in \mathcal{U}, \xi \in C_f} \{g(\xi) + \int_t^{T_f} l(\tau, x(\tau), u(\tau)) d\tau\}$   
with  $x(T_f) = \xi, x(t) = z, \dot{x}(\tau) = f(\tau, x(\tau), u(\tau)), \mathcal{L} - a.e..$

The value function is propagated by solving the Hamilton-Jacobi-Bellman equation (H-J-B pde). For details, check for example [Bardi and Capuzzo-Dolcetta, 1997]. In general, the value function is, at most, merely continuous, and, thus, the partial derivatives have to be understood in a generalized sense, and the solution concept has to be cast in a nonsmooth context, [Bardi and Capuzzo-Dolcetta, 1997; Clarke, 1996; Clarke et al., 1998]. The later will have to be adapted to the nature of the solution: in a viscosity, generalized, and proximal normal senses, for, respectively, continuous, Lipschitz continuous, and lower semi-continuous solutions. There are a number of software packages to solve the H-J-B pde numerically and thus compute the value function, see, for example, [Sethian, 1999; Michel et al., 2005; Mitchell, 2008; Cross and Mitchell, 2008] and the associated huge computational burden is well known.

In our framework, one or more value functions associated with a reasonable number of typified situations (which are strongly application dependent) are computed off-line and stored in a look-up table to be stored on-board the AUVs computers. During the execution of the “mission” in real-time, the relevant value function is identified via sensed data or by estimation from navigation data and invoked to determine the next optimal control at any point  $(t, x)$  that leads the AUV to the optimum of the recruited value function within the control horizon reach set.

Of course, situations may arise in which none of the anticipated typified situations occur. Then, two possibilities arise: either (i) the issue lies on “small” variations of parameters used in the typification process, or (ii) either the variations in (i) are large, or there occurs significant unexpected events, like, for example, emergence, of obstacles. In case (i), the value function is updated by an approximation constructed with a computationally simple linear interpolation. In case (ii), there are two possibilities: either the control architecture (see section 5) changes the mode of operation, or more sophisticated backward propagation using the current value of the adjoint variable computed by using the current conditions is used to propagate a first order approximation of the value function on the short term reach set. Details appear in ([Gomes, 2017]). whenever there are changes in the environment or in the system that affect the formulation of the underlying optimal control problem as it follows from the general requirements discussed above.

Now, we introduce the Reach Set for the control horizon to be used in our MPC scheme. Since the dynamics of robotic are time invariant, we can easily compute the points in the state space that can be reached in case there are no unexpected external interferences.

Let us define the forward reach set at time  $t$ , from the state  $x_0$  and time  $t_0 \leq t$ , [Graettinger and Krogh, 1991; Varaiya, 1998; Kurzanski and Varaiya, 2001], by

$$\mathcal{R}_f(t; t_0, x_0) := \{x(t) : \dot{x} = f(t, x, u), u \in \mathcal{U}, x(t_0) = x_0\}.$$

#### 4.2 The Reach Set MPC Procedure

Without any loss of generality, we proceed with a standard change of variable that eliminates the running cost to facilitate the presentation. Let  $\tilde{V}(t, \tilde{x}) = V(t, x) + y$  where  $\tilde{x} = (x, y)$ , being  $\dot{y} = l(t, x, u)$  with  $y(0) = 0$ . Without relabeling (i.e.,  $x = \tilde{x}$ , and  $V = \tilde{V}$ ), the optimal control problem ( $P_T$ ) can be expressed in terms of Reach sets and the value function as follows:

$$(P_T) \text{ Minimize } V(t+T, x(t+T)) \\ \text{subject to } x(t+T) \in \mathcal{R}_f(t+T; t, x(t)).$$

Let  $T$  be the optimization horizon,  $\Delta$ , the control horizon, and  $t$  the current time. Then, the MPC scheme can be formulated as follows:

1. Initialization:  $t = t_0, x(t_0)$
2. Solve ( $P_T$ ) over  $[t, t+T]$  to obtain  $u^*$
3. Apply  $u^*$  during  $[t, t+\Delta]$
4. Sample  $x$  at  $t+\Delta$  to obtain  $\bar{x} = x(t+\Delta)$
5. Slide time, i.e.,  $t = t+\Delta$ , update the Reach Set from the new  $x(t)$  by appropriate translation and rotation, update the value function at the new  $t+T$  if necessary, and goto 2.

It is clear in this scheme that the computational burden in real-time is extremely low, particularly, when comparing with the conventional MPC schemes. Under mild assumptions on the data and, by using the fact that the value function is continuous, it has been shown that this scheme is robust. Stability is proved by showing that there exists a uniform neighborhood along the reference trajectory for which there exists a control for which the value function satisfies a Lyapunov inequality in a generalized sense. From the continuity of the value function, we obtain sub-optimality estimates in both global and local senses as stated in the asymptotic performance convergence result below. To state this result, we require some notation. Let  $T$ , and  $\Delta$ , be, respectively, the optimization and control horizons, and denote by  $(x_{T,\Delta}^*, u_{T,\Delta}^*)$  the associated MPC optimal control process. Let  $J(x, u)$  be the value of the cost functional associated with the  $(x, u)$  over  $[0, \infty)$ , by  $J(x, u)|_{[\alpha, \beta]}$  be its restriction to the interval  $[\alpha, \beta]$ , and by  $J_k(x, u)$  its restriction to the interval  $[k\Delta, (k+1)\Delta]$ . **Proposition 1.** *Let  $T_f = \infty$  and assume that the optimal control horizon has an optimal control process  $(x^*, u^*)$  such that  $\lim_{t \rightarrow \infty} x^*(t) = \xi^*$ , being  $\xi^*$  an equilibrium point in  $C_\infty$ . Then,*

- (i)  $\lim_{\Delta \downarrow 0, T \uparrow \infty} \sum_{k=1}^{\infty} J_k(x_{T,\Delta}^*, u_{T,\Delta}^*) = J(x^*, u^*)$
- (ii)  $\lim_{k \rightarrow \infty} |J_k(x_{T,\Delta}^*, u_{T,\Delta}^*) - J(x^*, u^*)|_{[k\Delta, (k+1)\Delta]} = 0$ .

The simplicity of the optimization problem is apparent due to the complexity of the Reach Set computation. However, the invariance of the dynamics allows the pre-computation off-line of an approximation - either polygonal or pointwise - of  $\mathcal{R}_f(t_0 + T; t_0, x_0)$  as depicted in the figure 1 and store it in the AUV onboard computer.

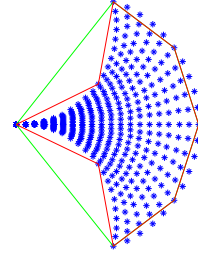


Figure 1. AUV forward reach set

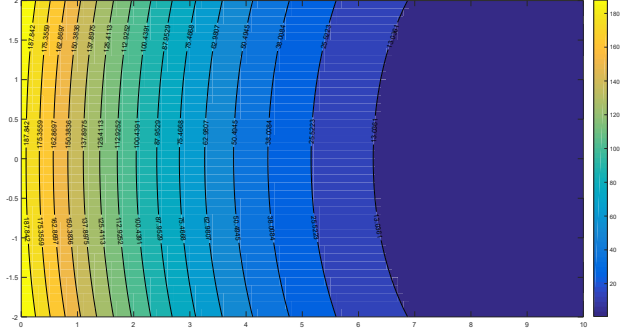


Figure 2. AUV value function computation example

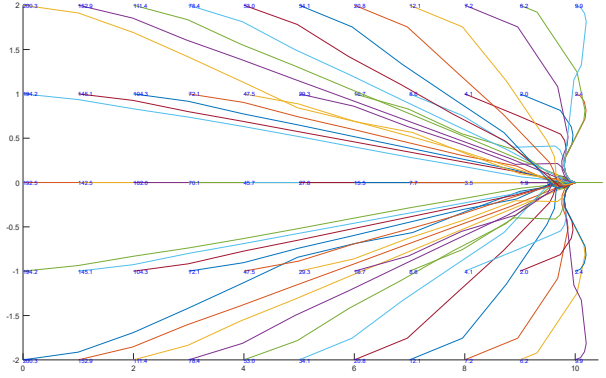


Figure 3. AUV optimal control lookup table example

Since the points of the state space that can be reached by the AUV from some reference position at a given time depend solely on the reference position and on the orientation at that time, the computation of the Reachable Set along the trajectory can be obtained by translations and rotations of  $\mathcal{R}_f(t_0 + T; t_0, x_0)$ .

As an example, we present in figure 2 the computed value function for an AUV in an area where the target point to which the system must be steered is  $(0, 10)$ . The value function was pre-computed by solving several off-line optimal control problems with different initial conditions spread across a state space partition as illustrated in figure 3. Here we can observe all the trajectories starting from the partition and converging to the final target. The value function for each trajectory is plotted at the beginning of the trajectory. For the sake of accuracy an interpolation to a thinner partition was calculated giving rise to the map in figure 2.

The next step involves overlapping the forward Reach Set with the value function map and finding the minimum value leading to the controls to be applied to the vehicle.

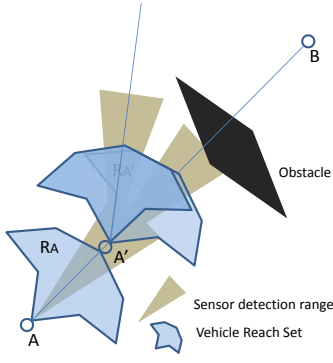


Figure 4. Reach set obstacle avoidance example

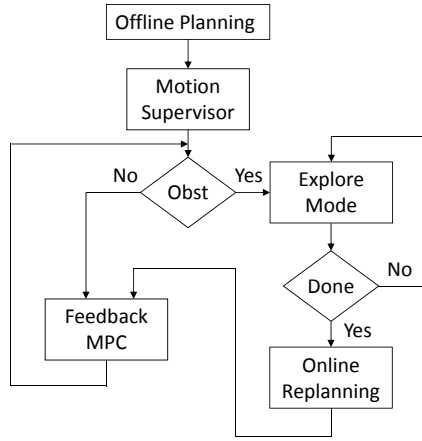


Figure 5. Reach set obstacle avoidance control architecture

## 5 The Control Architecture

Given the high variability of the environment due to expected and unexpected events, the overall control system requires to enter with an appropriate degree of situational awareness, being the articulation provided by the Control Architecture (CA). The issues concerning the perception system fall outside the scope of the current article, and, thus, we just illustrate the way the proposed MPC scheme, by taking into account onboard sensing devices, accommodates unexpected significant events in the context of the CA.

As depicted in figure 4, an obstacle is detected by the motion supervisor whenever it falls within the cone of the range sensor. In the CA diagram depicted in figure 5, it's clear that the AUV proceeds with its motion generated by the MPC scheme while the obstacle is not detected. Once this event happens, the motion supervisor switches to an exploration mode in order to find the optimal way to circumvent the obstacle by taking into account the original final target. In this process, the pre-computed value function is used and, once the exploration activity is successfully terminated, a new path from there on to the original final target is replanned. Mind you that, a priori, the value function in the state space region after circumvention of the obstacle remains exactly as the one pre-computed off-line. In figure 6, an illustration of an hypothetical behavior re-

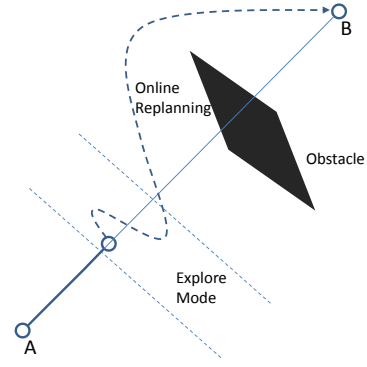


Figure 6. Reach set obstacle avoidance tracking example

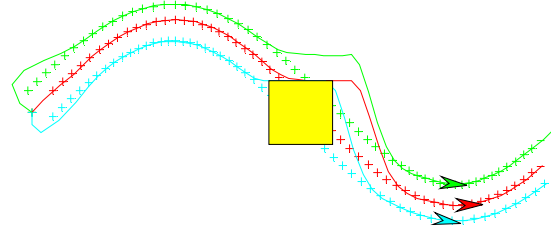


Figure 7. The overall simulation environment for AUV formation control with an obstacle

sulting from the CA is shown. A simple example illustrating the proposed scheme in controlling a triangle formation of AUVs which avoid the collision with an unexpected obstacle detected by their range sensors while mitigating the extent of the distortion of their formation pattern can be seen in figure 7.

## 6 Conclusions

In this article, we introduced a novel MPC scheme that enters into account the specific requirements that arise in the coordinated control of multiple AUVs. The key driver of the approach concerns the mitigation of the real-time computational burden for two strong reasons: onboard limited energy, computational power, and communication capabilities. The mathematical details were strongly omitted due to the lack of space. However, the simulation results obtained so far are extremely encouraging, being the next step the migration for this control structure for the AUV onboard control software for the required field testing.

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